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Comparison Of Resource Allocation Algorithms For OFDMA

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ABSTRACT
The report focuses on resource allocation algorithms (RAA) in Multi-user Orthogonal frequency division multiple access (OFDMA) Wireless System. The Resource Allocation algorithms are employed to solve the problems of margin and rate optimization in OFDMA. The algorithms are simulated and the parameters used for comparison are Jain's Fairness Index (FI), sum capacity and capacity distribution among a given number of users.
***Keywords* –** RAA, OFDMA, Fairness Index, Rate Optimization, Margin Optimization

I. INTRODUCTION

Resource management is crucial for OFDMA wireless broadband networks where scarce spectral resources are shared by multiple users. Resource management is usually separated into two parts: scheduling and resource allocation. Scheduling and resource allocation are essential components of wireless data systems. Here by scheduling we refer the problem of determining which users will be active in a given time-slot; resource allocation refers to the problem of allocating physical layer resources such as bandwidth and power among these active users. The resource allocation can also be divided into dynamic sub-channel assignment (DSA) and adaptive power allocation (APA).

The problem of assigning subcarriers and power to the different users in an OFDMA system has recently been an area of active research. For each user, bit and power allocation among sub-channels is an efficient method to exploit the frequency diversity inherent in frequency- elective channels. Over the past decade, the resource location problem for multiuser OFDMA systems has been extensively investigated. In particular, different sub-channel, power, and bit allocation schemes with diverse optimization objectives have been studied.

In this report, we will consider the resource management for OFDMA that offers performance gains needed for end-to-end QoS. We will focus on resource allocation algorithms and explore the fundamental mechanisms such as throughput, fairness, and stability. The organization of the rest of the report is as follows: in Section 5.2, we will introduce the margin and rate adaptive optimization. The different algorithms for (DSA) and (APA) will be reviewed in next sections.

II. System Model

We will consider multi-user OFDM systems with single transmit antennas. The base station will serve K users with only one receive antenna. The data are formed into OFDM symbols with N subcarriers and then transmitted through frequency selective channels. These channels are assumed to be constant over one OFDM frame and varying between the frames considering the Doppler frequency. Moreover, it is assumed that the channel taps are equal or smaller than the length of the cyclic prefix. Assuming that the CSI about all the subcarriers for all the users is known at the transmitter, the adaptive resource allocation algorithm is used to optimize the system parameters in a way that maximizes the total number of bits received by all the users for a given total power or minimizes the required total power for given user rate constraints. In this system, the channel that is between the base station and the kth user is described in the frequency domain as

$$H_k = [H_{k,1}\ H_{k,2} \ldots H_{k,N}]^T$$

where $H_{k,n}$ is the channel gain from the transmitter to the kth user for the nth subcarrier. In this scheme, the received signal is written as

$$Y_{k,n} = H_{k,n} S_{k,n} + N_{k,n}$$

where $S_{k,n}$ is the transmitted symbol with the power of P_T/N, and $N_{k,n}$ is the additive white Gaussian noise, with zero mean and variance of $N_0/2N$. In this system, the total bandwidth, B, is equally divided into N orthogonal subcarriers and the bandwidth of a subcarrier is equal to B/N.

Once the subcarriers have been determined for each user, the base station has to inform each user which subcarriers have been allocated to it. Therefore, it is assumed that the subcarrier/bit allocation information is transmitted to each user through a separate control channel. The resource allocation must be performed on the order of the channel coherence time. However, it may be performed more frequently if many users are competing for resources. The resource allocation is

usually formulated as a constrained optimization problem. Two different approaches are possible:
Margin adaptive (MA) optimization: Minimize the total transmit power with a constraint on the user data rate [1–3].
Rate adaptive (RA) optimization: Maximize the total data rate with a constraint on total transmit power [4–6].

The rate maximization optimization problem is an NP-hard combinatorial problem. Some algorithms have been proposed to solve this problem after relaxing some constraints, but the complexity of the algorithm becomes prohibitive for systems with a large number of subcarriers. Consequently, the most common way of solving the problem is to split it into two different phases:
DSA: The aim of this first phase is to assign the different sub-channels to the different users assuming the available total transmit power, P_T.
APA optimization algorithms: Once the sub-channel assignment is fixed, the APA must maximize the sum data rate given the different constraints.

III. Dynamic Sub-channel Assignment

DSA for RA optimization depends on the user rate constraints. In case there is no minimum user rate constraint (no condition C4), Jang and Lee [6] proved that the sum data rate is maximized when each sub-channel is assigned to the user with the best sub-channel gain assuming the power is shared equally between the sub-channels.
Description of the Jang and Lee algorithm:

- *Initialization*: For each user k, initialize the associated set of subcarriers allocated to the users, $Ck = \emptyset$.
- For $n = 1$ to N: k= arg $\max_k |H_{k,n}|$, the subcarrier n is allocated to the user k', $C_{k'} = C_{k'}\{n\}$.

In this case there is no fairness between the users and when the users have large path loss differences, the users with low average channel gains will be unable to receive data. Fairness requires a fair share of bandwidth among competing users. One of the representative types for the fairness is proportional fairness that provides each connection with a priority inversely proportional to its data rate.
The fairness index (FI) is calculated by using the Jain index [11] given.

$$FI = \frac{(\sum_{k=1}^{K} x_k)^2}{K\sum_{k=1}^{K} x_k^2}$$

where x_k can be equal to the allocated rate, rk , or the difference between the allocated rate and minimum required rate, $r_k - R_k$. (Note that if R_k is higher than r_k , x_k will be equal to zero.) The FI ranges between 0 (no fairness) and 1 (perfect fairness) in which all users would achieve the same data rate.

When a complete fairness is required between the users in the absence of the C4 condition, the DSA is performed to maximize the minimum user data rate under the power constraint assuming that the power is shared equally between the sub-channels. In Ref. [5], the authors proposed a reduced complexity suboptimal adaptive sub-channel allocation algorithm to solve this problem.
Description of the Rhee and Cioffi max–min algorithm:

- Initialization: For each user k, initialize $r_k = 0$. Set A = {1, 2, . . . , N}.
- For k = 1 to K :
  (a) n’ = arg maxn∈A $|H_{k,n}|$, the subcarrier n’ is allocated to the user k.
  (b) $r_k = r_k + (B/N) \log2(1 + SNR_{k,n'})$.
  (c) A = A − {n’}.
- While A ≠ ∅
  (a) k’ = arg $\min_k r_k$.
  (b) n’= arg $\max_n |H_{k',n'}|$, the subcarrier n is allocated to the user k’.
  (c) $r_{k'} = r_{k'} + (B/N) \log_2 (1 + SNR_{k',n'})$
  (d) A = A − {n’}

Channel swapping can be performed to maximize the max–min capacity but the initial algorithm already achieves a good performance. When there is a minimum user data rate constraint (condition C4), the dynamic sub-channel assignment can be solved in one step or divided into two tasks: bandwidth assignment and Sub-channel assignment.

3.1 Task 1: Bandwidth Assignment

In this task, the number *N*u of sub-channels per user is assigned. A greedy bandwidth assignment based on SNR (BABS) algorithm has been proposed for MA optimization in Ref. [2] and for RA optimization in Ref. [7]. In this task, we assume that all the sub-channels of a given user have the same gain. Let $|H_k|^2$ be the average user gain:

$$|\bar{H}_k|^2 = \frac{1}{N}\sum_{n=1}^{N}|H_{k,n}|^2.$$

Description of BABS algorithm:

- *Initialization*: Let $N_k = 1$ for each user k and $N_a = \sum^{K}_{k=1}$where N_k is the number of allocated subcarriers. $P_k(N_k)$ is the transmit power required by user k to achieve the data rate R_k using Nk subcarriers.

$$P_k(N_k) = \frac{N_k}{|\bar{H}_k|^2} f\left(\frac{R_k}{N_k(B/N)}\right)$$

- *Iteration:*
- If $\frac{\sum_{k=1}^{K} P_k(N_k)}{N_a} \le \frac{P_T}{N}$, stop; otherwise continue.
- While $\frac{\sum_{k=1}^{K} P_k(N_k)}{N_a} > \frac{P_T}{N}$

(a) Let $\Delta P_k = P_k(N_k) - P_k(N_k + 1)$ for $k = 1, 2, \ldots, K$.
(b) $k' = \arg\max_k \Delta P_k$.
(c) $N_{k'} = N_{k'} + 1$ and update N_a.

This algorithm gradually increases the number of subcarriers assigned to the users as *Nk* and gives the power value for assigned subcarriers as *Pk/Nk* . For BABS algorithm, it should be noticed that all the subcarriers are not necessarily allocated. In order to determine the number of subcarriers for a given QoS criterion, the bandwidth allocation on rate estimation (BARE) algorithm [8] is described as follows:

- *Initialization*: Let $N_k = N/K$ for each user k. We assume equal power allocation on all subcarriers, $p_{k,n} = P_T/N$. Then, compute the differences between the estimated rate of user k and the required minimum rate Rk :

$$G_k(N_k) = N_k C_k - R_k$$

where

$$C_k = \frac{B}{N}\log_2\left(1 + \frac{P_T}{N}\frac{|\bar{H}_k|^2}{N_0(B/N)}\right)$$

Iteration:

- While $\sum_{k=1}^{K} N_k < N$, find the user with minimum gap, $\kappa = \arg\min(G_k < 0)$. Then, the user κ receives one extra subcarrier.
- When Kk=1 $N_k = N$ and at least one predicted rate is less than the required minimum rate:
  (a) $k' = \arg\min_k(G_k(N_k) < 0)$ and provided that $k' = \arg\max_k(G_k(N_k - 1) > 0)$
  (b) $N_{k'} = N_{k'} - 1$ for user k' and $N_{k'} = N_{k'} + 1$ for user k_k.
- Continue until $G_k(N_k) > 0$ for all k.

When the power is too low to meet the common user rate guaranty, a fairness mechanism that decreases the users' rate constraints is applied before restarting BARE.

3.2 Task 2: Sub-channel Assignment

Once we determine the number of subcarriers allocated to each user, we perform the sub-channel assignment.

The sub-channel assignment is optimally solved using the Hungarian algorithm introduced by H. W. Kuhn in 1955. This problem is equivalent to the search of the optimum matching of a bipartite graph. The algorithm is briefly explained:

1. Find the minimum value of each row in the cost matrix R and subtract it from the corresponding row.
2. For the columns without zero, find the minimum value of the column and subtract it from the corresponding column.
3. Cover the zeros with the minimum number of horizontal and vertical lines in the updated cost matrix. If the minimum number of lines equals the dimension of the matrix, then stop. Else go to step 4.
4. Find the minimum value in the uncovered part of the cost matrix, and subtract it from the uncovered elements. Add it to the twice covered elements (elements at the intersection of a horizontal and a vertical line). Return to step 3.

The ACG algorithm was initially proposed by Kivanc et al. [2] and an improved version has been proposed in Ref. [9]. Description of the improved ACG is given below:

Initialization: For each user k, initialize the associated set of subcarriers allocated $Ck = \emptyset$. Set $A = \{1, 2, \ldots, N\}$.

Allocation: For $i = 1$ to N:

$$k', n' = \arg \max_{n\in A \quad k:card(C_k)<N_k} r_{k,n}$$

the subcarrier $n\_$ is allocated to the user k': $A = A\backslash\{n'\}$and $C_{k'} = C_{k'}\{n'\}$

It should be noticed that the improved ACG algorithm can perform subcarrier assignment when the total number of allocated subcarriers is less than N.

3.3 Combined Tasks 1 and 2

In order to combine Tasks 1 and 2, in Ref. [3], a reduced complexity subcarrier and bit allocation algorithm has been proposed assuming equally shared power between subcarriers. The complexity of this algorithm has been significantly reduced by selecting the initial solution as an unconstrained optimal one. Besides that, only one constraint needs to be considered during each searching stage.

The Zhang and Letaif algorithm is described briefly as follows.

Step 1: Optimization without inequality constraints: Firstly, the bit and subcarrier allocation is done without considering the rate constraint for each user as described in Jang and Lee algorithm.

Step 2: Subcarrier reallocation: The subcarrier allocation solution from Step 1 does not guarantee the fulfillment of every user's rate constraint. This subcarrier reallocation process is repeated until all the user's data rate requirements are satisfied. During the reallocation process, the following conditions must be satisfied:

- A subcarrier that was originally assigned to user $k* n$ cannot be reallocated to another user if the reallocation will cause the violation of user $k* n$ s data rate requirement
- Each subcarrier reallocation should cause the least possible reduction in the overall throughput
- The number of reallocation operations should be kept as low as possible.

IV. APA Algorithms

Once the sub-channel assignment has been performed, the APA is performed to maximize the sum data rate *r* according to the given total power constraint. In the absence of condition C4, APA algorithm can be treated as a virtual single user OFDM system. Otherwise, it should be applied separately for each user. The Zhang and Letaif algorithm is described in Figure 1 in detail.

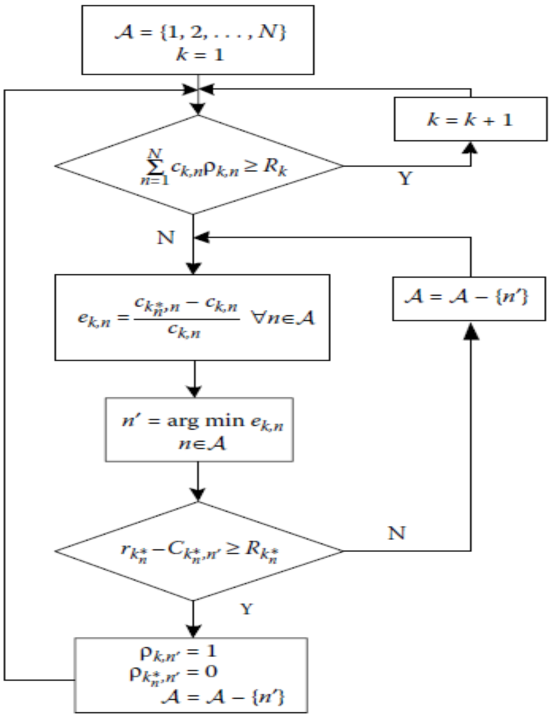

Figure 1: Subcarrier reallocation in Zhang and Letaif algorithm.

In practice, continuous rate adaptation is not feasible, and there are only several modulation levels. Consequently, the water-filling algorithm cannot achieve the optimal power allocation. For discrete modulation levels, a greedy power allocation algorithm has been proposed. The key idea of the greedy algorithm is to allocate bits and the corresponding power successively and to maximize the partial sum data rate in each step of bit loading [3]. In initialization, zero bits are assigned to all subcarriers. During each bit loading iterations, the subcarrier that needs the minimum additional power is assigned one more bit and the total partial power is updated. The iteration process will stop when the total transmission power constraint is reached. To apply the greedy power allocation for discrete rate adaptation, the modified Levin–Campello algorithm [10] can be used and it is described as follows:

4.1 The Modified Levin–Campello Algorithm

Let $\Delta P_n(c)=(f(c+1)-f(c))/|H_n|^2)$ denote the incremental power needed for the transmission of one additional bit at subcarrier *n*, and *c* is the number of loaded bits for the *n*th subcarrier.

- *Initialization*: For each subcarrier *n*, initialize $c_n = 0$ and evaluate $\Delta P_n(c = 0)$ with tentative transmit power $P_T^* = 0$.
- *Bit assignment iteration*: Repeat the following iterations until $P_T^* \geq PT$:
  $n* = \arg\min_n \Delta P_n(c_n)$
  $P_T^* = P_T^* + \Delta Pn^*(cn^*)$
  $cn^* = cn^* + 1$
  if $cn^* = c\max$, set $\Delta Pn^*(cn^*)=\infty$, else evaluate $\Delta Pn^*(cn^*)$.
- *Finish*: The allocation result $\{cn\}_{n=1}^{N}$ is the obtained optimal bit allocation solution.

V. Results

We evaluate the different resource allocation algorithms using the parameters listed in Table 1.

Table 1: System Parameters for the Simulation

Parameters	Value
Cell radius	1.6km
BS transmit power	43.10dBm
Noise power	−174dBm
Path loss *Lp*	$128.1+37.6\log_{10}(d)$dB
Channel model	3GPP TU
Number of clusters	48
Bandwidth	10Mhz
Carrier frequency	2.4Ghz
Velocity	3 km/h
Simulation time	5 s
User distribution	[0.3 0.4 0.5 0.8 1.0]km, equal probability

The sum capacity comparison results are obtained for different allocation algorithms of OFDMA systems in Figure 2. According to the results, the best sum capacity performance is obtained using the Jang–Lee algorithm and the Zhang–Letaif algorithm. However, the FI is also an important parameter to observe the distribution of the users' data rate. In Figure 3, the Jain FI is drawn and we can see that the Jang–Lee algorithm does not provide fairness. The Zhang–Letaif algorithm can bring limited fairness without sacrificing the sum capacity performance. By construction, the Rhee–Cioffi algorithm maximizes the fairness between the users when equal rate constraints are considered for all users. The BARE+Improved ACG algorithm gives a slightly better sum capacity performance than the

Rhee–Cioffi algorithm and provides fairness compared to the Zhang–Letaif algorithm.

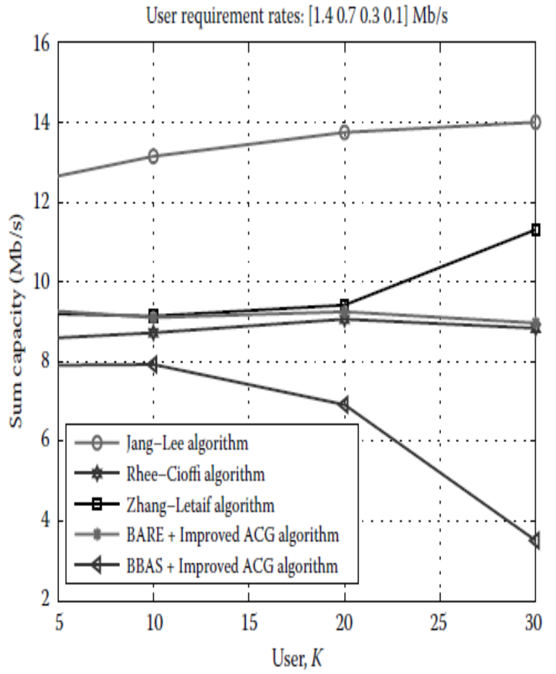

Figure 2: Sum capacity versus the number of users.

The BABS algorithm is based on subcarrier assignment that includes power loading; compared to the BARE algorithm, the BABS algorithm also provides fairness. It should be noticed that the BABS algorithm allocates the minimum number of subcarriers to satisfy the users' requirements and the total number of allocated subcarriers is not always equal to the number of total subcarriers in OFDMA, while the BARE algorithm fulfills all the subcarriers to increase the sum data rate performance.

In Figure 4, the distribution of the users' data rate is shown for K =5 users in the systems. It is shown that the Zhang–Letaif and Rhee–Cioffi algorithms distribute the available resources and the capacity between the users and satisfy some users' QoS constraints, while the Jang–Lee algorithm does not consider the users' rate constraints. It is observed that the resource allocation based on subcarrier assignment using the BABS and BARE algorithms achieves all the users' rate requirement.

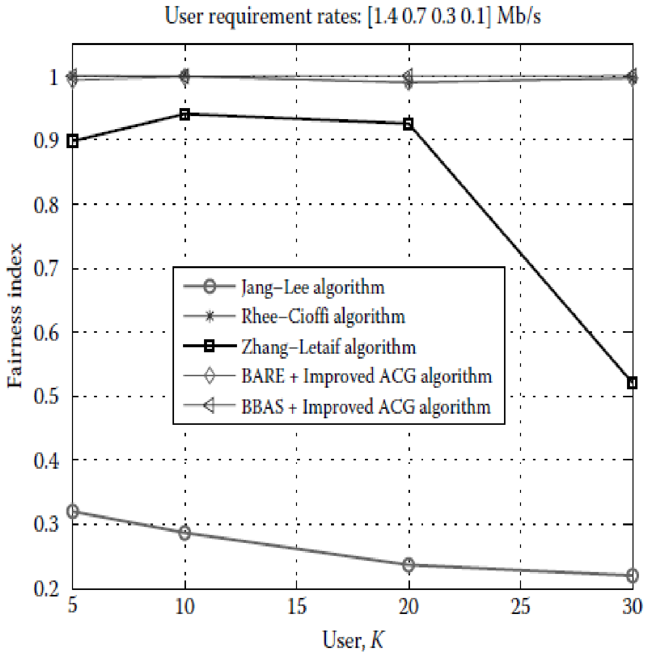

Figure 3: FI versus the number of users.

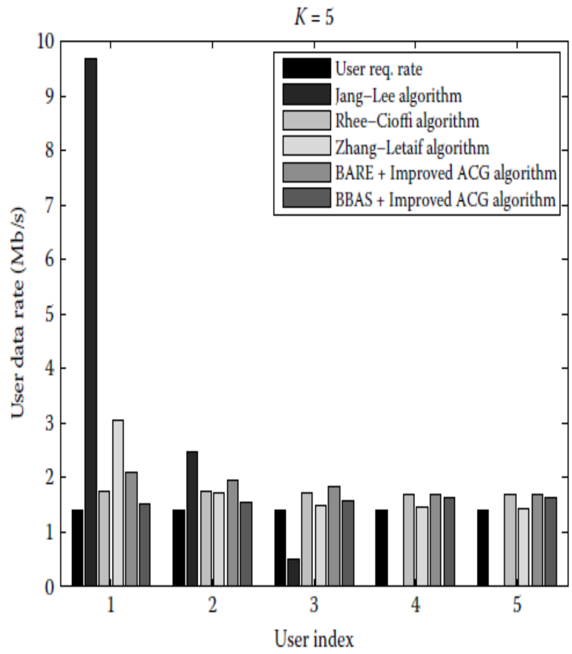

Figure 4: Capacity distribution among the users for $K = 5$.

VI. Conclusion

In sum, the resource allocation algorithm are studied and simulated. The BARE and BBAS algorithm have the best fairness index compared to all. The Jang–Lee algorithm has the best sum capacity among all algorithms. Zhang–Letaif and Rhee–Cioffi algorithms distribute the available resources and the capacity between the users and satisfy some users' QoS constraints and the BABS and BARE algorithms achieve all the users' rate requirement. The algorithms have solved the RA and MA optimization problem.

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